SUBJECTIVE REFRACTION: A NEW VECTORIAL METHOD FOR DETERMINING THE CYLINDER (1/3)

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Today, thanks to phoropters with continuous power changes that allow to simultaneously and accurately act on sphere, cylinder and axis, it is now possible to develop new refraction techniques. This series of three articles describes the principles of a new vectorial method for determining the corrective cylinder and presents the rationale for an associated automated cylinder search algorithm.

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KEYWORDS
Subjective refraction, vectorial refraction, dioptric space, cylinder search, cross cylinders, phoropter, refraction algorithm, Vision-R™ 800.

For nearly a century, the refraction technique used to determine a patient's corrective cylinder has remained almost totally unchanged, mainly because subjective phoropters themselves have changed very little. Practitioners generally use the Jackson cross-cylinder method, studying the variation of its effects for different positions, to determine first the cylinder axis, then the cylinder power and, finally, to adjust the effect on sphere power. With a subjective phoropter, practitioners present spherical and cylindrical lenses in front of the patient's eye in increments usually not smaller than 0.25 D and 5 degrees in axis. Simultaneous action on the sphere, cylinder and axis is also not possible.

Today, the advent of phoropters that offer continuous power changes – with a resolution of 0.01 diopter and 0.1 degree – and allow to act on sphere, cylinder and axis all at the same time(*) makes a new approach to subjective refraction possible: it is called "Digital Infinite Refraction™"[1]. A vectorial method has been developed to determine the cylinder that is both more consistent and more accurate.

This series of three articles provides an overview of this new vectorial method. In this first article, we will review the vectorial definition of refraction and its representation in the 'Dioptric Space' before offering a general comparison of the "Traditional Refraction" and "Digital Infinite Refraction™" methods. The second article will describe in detail the techniques used in "Traditional Refraction" and "Digital Infinite Refraction™" to determine cylinder axis and cylinder power. The third and final article will present the new method of determining the cylinder made possible by "Digital Infinite Refraction™" in comparison with to the "Traditional Refraction" method, and will discuss its application to the development of an automated algorithm for determining the cylinder.

Read on to learn more about this new vectorial method for determining the corrective cylinder. Please note you will need to be familiar with the basic principles of refraction to fully understand these articles.

(*) Vision-R™ 800 phoropters with continuous power changes, Essilor Instruments"

Vectorial representation of the cylinder in a dioptric space

"Polar" vs "Cartesian" expression of a refraction:

Although in ophthalmic optics, the formula of a refraction is traditionally expressed with reference to its "Polar" expression (sphere, cylinder and axis), it is also possible to give it a "Cartesian" expression in the form of three coordinates:

1) the spherical equivalent or Mean sphere *M*, equal to the sphere power augmented by half of the cylinder power,
2) the cylinder component along the horizontal axis at 0° (J0°), representing the direct/indirect component of astigmatism,
3) the oblique component of the cylinder along the oblique axis at 45° (J45°), representing the oblique component of astigmatism.

The advantage of this cartesian expression is that it expresses the refractive formula in the form of three independent components, themselves expressed in a single and consistent unit: diopters. These can effectively replace the components of the traditional polar expression of a refraction (sphere, cylinder and axis), which are interdependent and expressed in different units: diopters for sphere and cylinder and degrees for the axis. The cartesian expression yields a unique global formula for a refraction that facilitates its analysis and statistical comparisons.(2)

By way of illustration, Table 1 shows examples of refraction formulas expressed in traditional polar coordinates transposed into cartesian coordinates. We can see that the cartesian expression of a refractive formula involves expressing the refraction in the form of an average component and two pure cylindrical components, which is to say similar to Jackson cross-cylinder formulas with null mean sphere power, one of them at 0°/90°, representing the horizontal/vertical component of the astigmatism, and the other at 45°/135°, representing its oblique component.

The relationship between the polar and cartesian expressions of a single refraction formula is based on a simple trigonometry calculation. It is relatively easy to move from one expression to the other:

– If we know the traditional polar formula of a refraction Sph (Cyl) Axis, we can calculate the three coordinates of its Cartesian expression using the following formulas:
  - *M = Sph + Cyl / 2* ;
  - *J0° = Cyl * Cos (2 * Axis)* ;
  - *J45° = Cyl * Sin (2 * Axis)*.

Because of the non-trigonometric cycle of the axis (its variation from 0 to 180° rather than 0° to 360°), it is necessary to double the value of the cylinder axis.

– Inversely, if we know the cylinder's cartesian components, J0° and J45°, it is easy to determine its polar (cylinder and axis) components via vectorial composition. And for the sphere, all we need to do to find its value is algebraically subtract half of the cylinder's value from that of the spherical equivalent. The formulas are as follows, using a negative cylinder convention:
  - *Sph = M – Cyl / 2*
  - $Cyl = -\sqrt{J0°^2 + J45°^2}$
  - *Axis = 0.5 * ArcTan (J45° / J0°) + C*, with C constant equal to 90 if J0° > 0 and equal to 0 if J0° < 0.

To make it easier to grasp and simpler to represent visually, we have opted in this article not to keep the ½ weighting between the values of the J0° and J45° components, on the one hand, and the *M* spherical equivalent power on the other hand, as is generally the case in the literature on vectorial expressions of refraction. The principle remains the same but this simplification is more readily understandable.

Representation of a prescription in a "Dioptric Space":

The advantage of the cartesian expression of a refraction is that it can represent any refractive formula in a three-dimensional orthogonal system called the "Dioptric Space". Any prescription is represented in it by a unique vector whose projections on the system three axes are the cartesian coordinates of the refractive formula.
As a result, the following is shown on the three axes:
– the spherical equivalent power, or mean sphere *M*,
– the horizontal component of the cylinder J0°,
– the oblique component of the cylinder J45°.

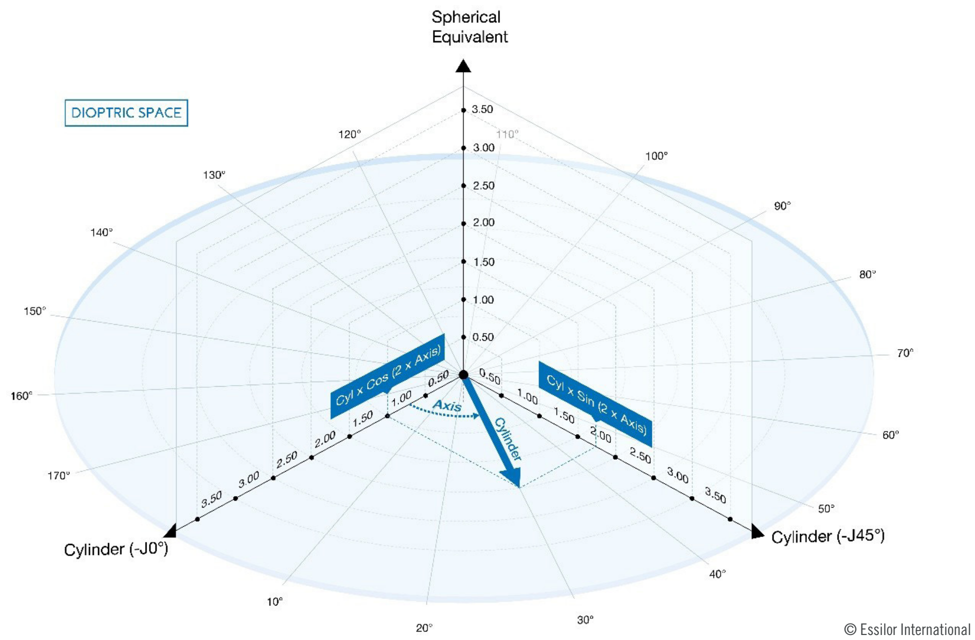
Table 1: Polar and Cartesian expressions of various refraction formulas

POLAR EXPRESSION			CARTESIAN EXPRESSION		
Sphere	**Cylinder**	**Axis**	**M**	**J0°**	**J45°**
+2.00			+2.00	0.00	0.00
-2.00			-2.00	0.00	0.00
Plano	-2.00	0	-1.00	-2.00	0.00
Plano	-2.00	90	-1.00	+2.00	0.00
Plano	-2.00	45	-1.00	0.00	-2.00
Plano	-2.00	135	-1.00	0.00	+2.00
+1.00	-2.00	120	0.00	+1.00	+1.73
+1.00	-2.00	30	0.00	-1.00	-1.73

The three-dimensional representation of the dioptric space we use, which allows us to easily visualise the characteristics of any refractive formula in 3D (see Figure 1), is a modified version of the conventional representation explained in more detail in various reference publications.[2,3,4,5] The sphere is expressed along the vertical axis and the cylinder along the horizontal plane : the cylinder axis is represented by the rotation around the vertical axis and the cylinder power by the distance from the origin, here chosen according to the negative cylinder convention. This model can be used to simply depict any refractive formula in the form of a single vector in the space and to study its variations during a refraction examination: the purpose of "Vectorial Refraction".

Figure 1: Vectorial representation of refraction in a dioptric space.

a) Cartesian coordinates: example of a refraction formula of +1.00 (-2.00) 30°



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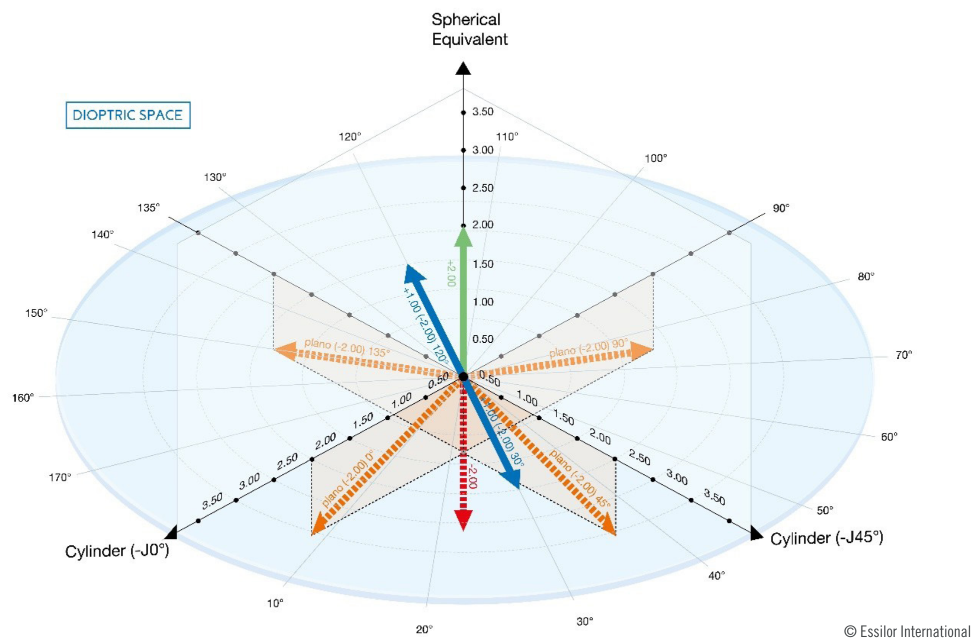
b) Examples of vector representations of different refraction formulas (presented in Table 1):



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Sphere formulas: +2.00 (in green) and -2.00 (in red);
Astigmatic formulas: plano (-2.00) with cylinder axes of 0°, 45°, 90° and 135° (in orange)
and +1.00 (-2.00) with cylinder axes at 30 ° and 120 ° (in blue).

The example we will be using in the rest of this article, a refraction formula of +1.00 (- 2.00) 30° with a null spherical equivalent, was chosen for the convenience of the graphic representations, since the corresponding vector is located on the horizontal J0°/J45° plane. For any other refraction whose spherical equivalent power is not null, the approach would be the same but the vector would move in the space, leaving a trace identical to that made on the J0°/J45° plane but on a parallel horizontal plane, corresponding to the value of the spherical equivalent.

Traditional refraction vs Digital Infinite Refraction™: similarities and differences

Although the traditional and digital refraction techniques have a few principles in common, they differ greatly in other points. Let us take a look at these similarities and differences before examining them more closely in the following two articles.

Refraction with "presentation of lenses" vs refraction with "continuous power changes"

- The "traditional" refraction technique involves presenting spherical and cylindrical lenses in front of the patient's eye. This can be done with trial frames and trial lenses, using a manual phoropter with mechanical lens changes or an automated phoropter with motorised lens changes. Regardless of the instrument used, the method involves presenting lenses in 0.25 D increments; only the way the lenses are changed is different. Furthermore, the sphere power, cylinder axis and cylinder power must be examined separately, one after the other, during the examination.

- The "digital" technique, on the other hand, takes advantage of the capacities of an optical module with continuous power changes(*) controlled by micro-motors with digital commands. This technology allows to switch instantly from one optical formula to another by modifying the optical powers and using the variation increment desired (with a resolution of 0.01 D). It is also possible to change the sphere power, cylinder axis and cylinder power simultaneously, allowing to move from one corrective formula to another with no delay. This property is what makes the new refraction technique possible.

Determining the refraction components "successively" vs "simultaneously"

- "Traditional" refraction techniques involve first determining the sphere and then the cylinder axis and power before finally adjusting the sphere. For the cylinder determination, it is important to always start with the cylinder axis before moving on to the cylinder power, otherwise the latter value will be impossible to determine correctly. While it is possible to adjust and find the correct value of a cylinder axis if its starting power is not correct, adjusting the power of a cylinder with an incorrect starting axis leads to a value different from that which would have been obtained with the correct axis.

- In the "digital" refraction technique, we firstly look for the mean sphere and then, in the same sequence, move on to the cylinder power and axis, keeping the spherical equivalent power exactly constant with a resolution of 0.01 D. Two refraction components are considered here: a power component along the initial axis of the starting correction and an axis component that is perpendicular to the latest in the dioptric space. Since these power and axis components are orthogonal and independent of each other, cylinder seeking can begin with either the axis or the power component. That said, the initial refraction measurements provided by autorefractometers are generally more accurate in the axis value than in the power value. This is why cylinder power is the starting point for the new digital refraction technique, unlike the traditional method which begins with seeking the axis.

Determining astigmatism: "physical" vs "virtual" cross cylinders

Both cylinder determination techniques ("traditional" and digital refraction) use the Jackson cross-cylinder method, named after the American ophthalmologist who developed it in the early 20th century.
Remember that the cross cylinder is a spherical-cylindrical lens resulting from a combination of two plano-cylindrical lenses with identical powers but opposite signs positioned perpendicularly to each other (this is the reason for the name "cross cylinders") and with a null spherical equivalent. Determining the corrective cylinder involves placing the cross cylinder in front of the patient's eye while they are wearing their correction and studying the variations in the sharpness of the patient's vision that result from the combination of the residual astigmatism of the eye + lens system and that of the cross cylinder at different positions.

Although this cross-cylinder method is similar in both refraction techniques, the approaches used are very different.

- In traditional refraction, physical cross cylinders in the phoropter are flipped over during the examination. Cross cylinders of +/-0.25 D or +/-0.50 D are generally used; their respective optical formulas are +0.25 (-0.50) and +0.50 (-1.00). Due to its construction, the "handle" of any cross cylinder bisects the axes of its positive and negative cylinders in such a way that, by simply flipping them over, one can switch their positions or, in other words, instantaneously turn the axis of the cross cylinder by 90° without modifying the mean sphere value. Practitioners use this property to look for the cylinder axis and power, seeking the orientation of the axis and then the value of the power at which turning over the cross cylinder produces an identical blurred vision for the patient. We will look at this technique in more detail in article two.

- In "digital" refraction, an optical principle similar to the Jackson cross-cylinder method is used but no cross cylinders are physically present in the phoropter. Optical cross-cylinder effects are generated in the optical module using calculations in combination with the existing correction. There is therefore no positioning of a cross cylinder in front of the patient's eye nor any interruption in their vision during the switch, only seamless changes in optical correction that the patient perceives instantly. The cross-cylinder power is not limited to that of a traditional cross cylinder (of +/- 0.25 D or +/- 0.50 D) but can be chosen with a resolution of 0.01 D to allow for easy comparison between the two positions and configuration during the design of the cylinder determination algorithm. It could also be adjusted during the refraction examination according to the patient's sensitivity. This flexibility offers remarkable possibilities in terms of improving and adapting refraction methods. In the example we have been using in this article, the cross-cylinder power is +/- 0.35 D.

Later, in article two, we will examine in detail the practical implementation and differences of these techniques when it comes to determining the cylinder.

An "unchangeable" traditional technique vs an "upgradeable" digital technique

– In "traditional" refraction, the testing technique and method for determining the cylinder have remained the same for the past century and there is little room for any change due to the physical limitations and mechanical constraints imposed by the instruments. The refraction is entrusted entirely to practitioners, who apply the knowledge they have acquired, their experiences and the type of approach they have chosen. As a result, there are inevitably variations among refraction results.

– In "digital" refraction, on the other hand, the testing and refraction methods used are innovative and upgradeable. Because the optical module is controlled by calculations and totally flexible, a wide field of possibilities opens up for the development of new refraction methods. The first refraction determination assistance algorithms have been invented to formalise the first examination principles. They should be able to bring about a certain standardisation in refraction methods. These algorithms are already "adaptive" that is, they have the capability to adapt to patients' answers during the examination itself. They will undoubtedly be improved upon as advances in this area are made, making many refraction assistance solutions possible in the future. The new "Digital Infinite Refraction™" approach therefore holds considerable potential for ongoing improvements in refraction methods.

We will continue the presentation and discussion of this topic in two articles to come.

KEY INFORMATION:

- The cylinder search technique has changed very little since Jackson's invention of the "cross-cylinder" method in the early 20th century because subjective phoropters whose functioning is based on a presentation of various lenses have themselves not changed much.
- Today, with the advent of phoropters offering continuous power changes, it is now possible to offer a new cylinder search method based on a vectorial approach to refraction.
- This method explores the "dioptric space" in a more direct way, searching for the cylinder power and axis simultaneously while keeping the spherical equivalent power exactly constant.
- Combined with the properties of a very precisely controlled optical module that is integrated into refraction search algorithms, this new technique offers great scope for advancements in refraction methods.

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KEYWORDS
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Following up on our first article, published in Points de Vue in November 2020, we continue our discussion of a new vectorial method for determining the cylinder. This second article compares the techniques used in "traditional" refraction with a new "digital" refraction method for determining the cylinder axis and cylinder power during a refraction examination.

3) Determining the cylinder: "Traditional Refraction" vs "Digital Infinite Refraction™"

In "traditional" refraction, the cylinder axis is always determined before the cylinder power. Let us take a look at each of them, comparing the "traditional" and "digital" methods for testing the axis and power.

a) Cylinder axis test:

- ***With "traditional refraction" technique***

The Jackson cross-cylinder technique is the most universal method for determining the cylinder axis of a correction. To do so, the practitioner places the handle of the cross-cylinder according to the direction of the axis of the corrective cylinder to be tested and offers the cross-cylinder to the patient in two positions by flipping it over. The combination of the cross-cylinder power and the residual astigmatism, resulting from the patient's eye and the correction in place, creates a perception of blurriness for the patient. The position of the cross-cylinder that the patient perceives to be less blurry indicates the direction that the axis of the correction should be adjusted in. In this way, with a succession of approaches, the practitioner searches for the position for which the patient perceives no difference in blurriness between two positions; the handle's orientation then indicates the direction of the corrective axis. More details on this traditional refraction technique can be found in a number of reference works.[6]

(*) Vision–R™ 800 phoropter with smooth power changes by Essilor Instruments

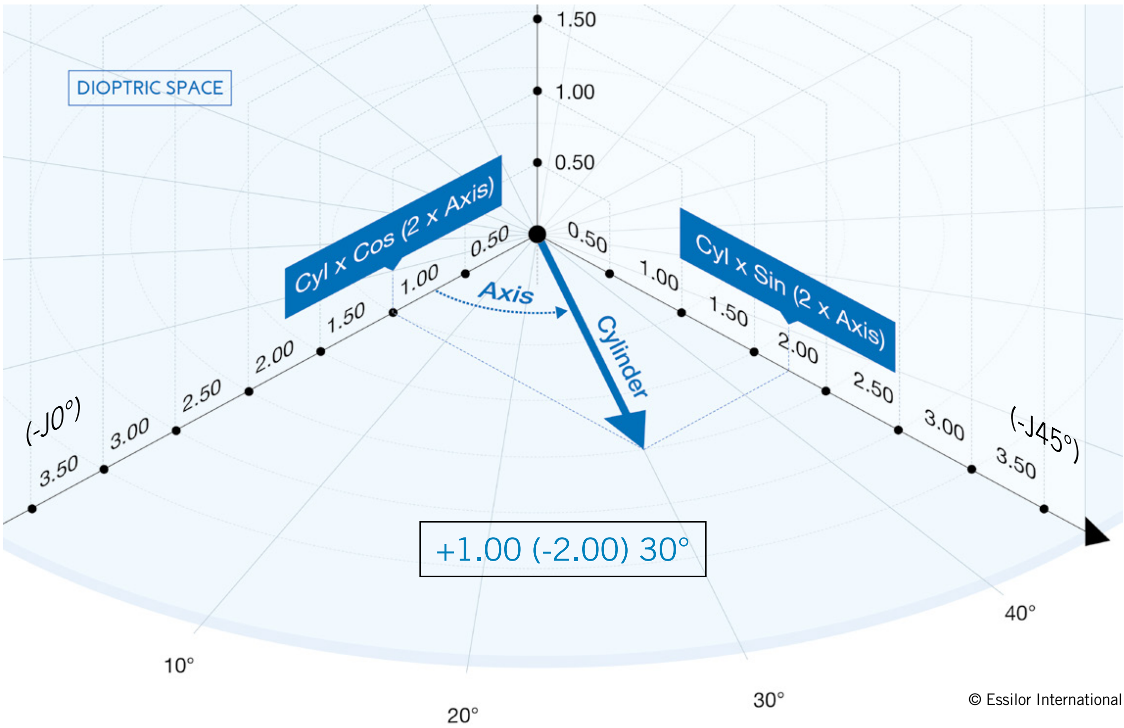


Figure 2: Vectorial representation of refraction in a Dioptric Space.
Cartesian coordinates: example of a refraction formula of +1.00 (-2.00) 30°

If we consider the example of Figure 2, showing a prescription of +1.00 (-2.00) 30°, the handle of the cross-cylinder is oriented to 30° and two positions of the cross-cylinder are being tested (see Figure 3): Position 1, with the negative axis of the cross cylinder at 165° (30° - 45° modulo 180°) and Position 2, with this same axis positioned at 75° (30° + 45°). The practitioner thus tests the results of two combinations of the cross-cylinder with the correction in place, for which the formulas are as follows: Position 1, +1.03 (-2.06) 23°, and Position 2, +1.03 (-2.06) 37°, i.e. for the example of a 2.00 D cylinder, an axis variation of 7° on either side of the corrective cylinder axis being tested (see Table 2). The patient then indicates which Position he/she prefers or, more specifically, which one is less blurry. Let us suppose that he/she prefers the second position. Traditionally, the practitioner then rotates the axis of the correction 5° and the cross-cylinder in the direction indicated, taking them to 35°, and performs the test again in the same way. He/she offers two combinations that are identical to the previous ones, with the resulting axis again situated at 7° on either side of the new axis direction tested, namely 35° tested, or +1.03 (-2.06) 28° for Position 3 and +1.03 (-2.06) 42° for Position 4. They continue in this way until the patient no longer perceives any difference between the two positions or asks to go back to an earlier axis direction.

At this point, we can make the following observations:

– In the "dioptric space", the effects of the cross-cylinder during the axis test are expressed perpendicularly to the direction of the vector representing the correction being tested, with a 0.50 D variation on either side (see Figure 3). In this test, the spherical equivalent power remains constant, since the spherical equivalent power of the cross-cylinder is null. Thus, any search for the axis takes place on the cylinder plane, J0° / J45° (or on a parallel plane if the spherical equivalent power of the chosen correction was not null).

– It is clear that when one is testing the axis of a corrective cylinder using a cross-cylinder, one is actually testing the effect that the cross-cylinder power induces on the axis of the resulting cross-cylinder + corrective cylinder when the cross-cylinder is positioned at 45° on either side of the axis of the corrective cylinder. In the example of a cylinder of (- 2.00) at 30°, one tests the effect of a +/- 7° variation in the axis on either side of the 30° direction, in other words 23° and 37°, caused by a 0.50 D cylinder oriented at +/- 45° with respect to the 30° axis (165° and 75° respectively).
For other cylinder power values, one would test other angle values: a few examples can be seen in Table 2, which presents tested axis variations with cross cylinders of +/- 0.25 D and +/- 0.50 D according to the cylinder power. We can see that the tested angle effect, expressed in degrees, is inversely proportional to the cylinder's value, which is perfectly consistent with the fact that patients are especially sensitive to cylinder axis variations when the corrective cylinder power is higher. But this effect, expressed in dioptric terms, remains constant because it is the value of the cross cylinder used (here, 0.50 D), which guarantees uniformity of perception in the optical effects observed by the patient during the search for the cylinder axis.

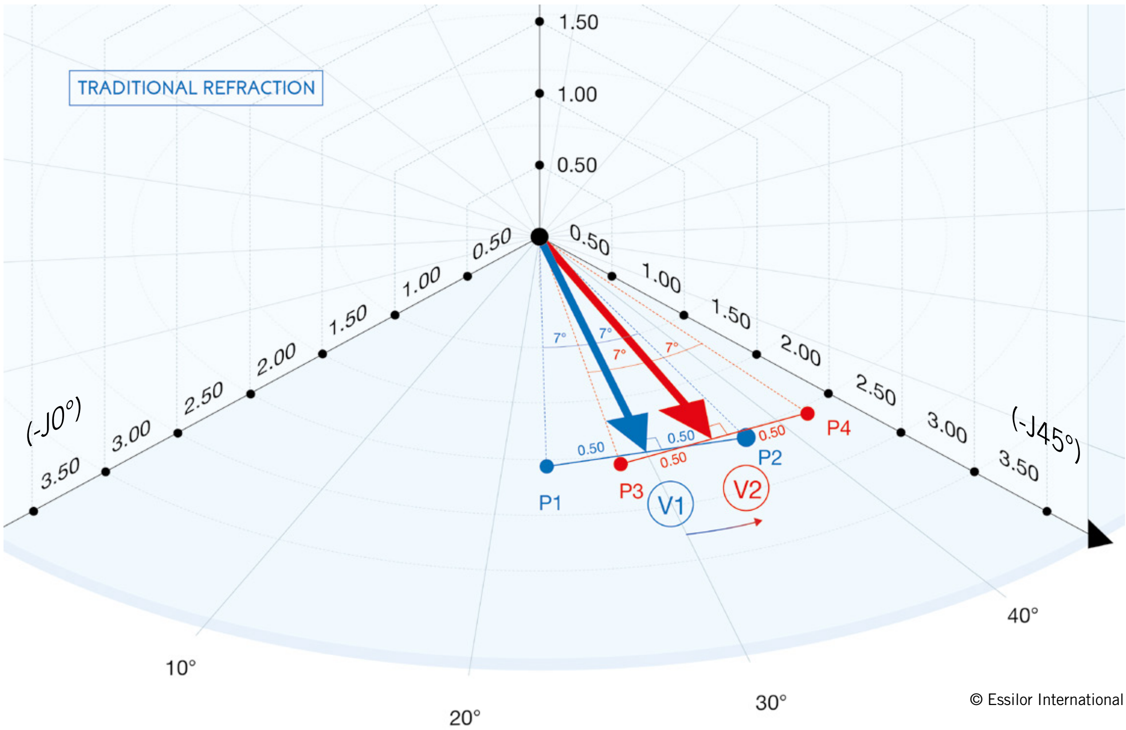


Figure 3: Cylinder axis test in the "Traditional Refraction" technique

Table 2: Tested axis variations with cross cylinders of +/– 0.25 D and +/– 0.50 D. Depending on the powers of the cross cylinder and cylinder, it is expressed in the formula: *Tested Axis Variation = ½ * Arc Tan (Cross Cylinder Power / Tested Cylinder Power)* ie, the values presented in the following table.

Power Cylinder Tested (Diopters)	Tested Axis Variations (Degrees)	
	With CC +/- 0.25 (Cyl 0.50)	Avec CC +/- 0.50 (Cyl 1.00)
0.50	+/- 22.5 °	+/- 31.7 °
1.00	+/- 13.3 °	+/- 22.5 °
1.50	+/- 9.2 °	+/- 16.8 °
2.00	+/- 7.0 °	+/- 13.3 °
2.50	+/- 5.7 °	+/- 10.9 °
3.00	+/- 4.7 °	+/- 9.2 °

– Once the first axis test is made based on the initial direction, the initial correction and cross-cylinder are turned together, for example another 5°, in the direction indicated by the patient in order to re-test the axis in a second direction. At this stage, several effects appear:

1) The effect observed by the patient during an axis test is not completely respected during the following tests. Remember that a cylinder can be considered a lens producing "a given power, at a given axis" and that, as a result, modifying the cylinder axis will modify its corrective optical effect and the patient's perception of it. During the rotation of the corrective cylinder axis between two axis tests with the cross-cylinder, for example by 5°, the cylinder power remains constant. Represented in the dioptric space (Figure 3), the cylinder's axial component is modified without its power component being adjusted, and the cylinder axis rotation, without adjustment to its power, does not keep the test direction of the cylinder's axial component constant. As a result, it does not respect the perception that the patient had of it during the previous axis test: the test conditions are thus modified with each axis rotation. In a certain sense, the information that the patient provides during the first axis test is not fully conserved and respected during the second axis test and other tests to follow. The various answers the patient gives under different conditions lack consistency and it is not possible to accumulate them and capitalise on previous answers to fine-tune the search for the cylinder axis and determine it accurately.

2) The axis system of reference in the dioptric space changes throughout the axis search process. After the 5° axis rotation, the second axis test is performed with reference to the new cylinder direction, in a new combination with a cross-cylinder oriented to 45° on one side (Position 3) of this direction and the other (Position 4). In the dioptric space, this test is carried out in a direction perpendicular to the new cylinder direction, which has been turned 5° and therefore in a direction that has also been turned 5° compared to the previous test direction. The system of reference is thus modified compared to the one used during the first cylinder test. It will then be modified with each change in the orientation of the cylinder axis tested throughout the process of determining the cylinder axis. As the system of reference has changed with each axis direction modification, the axis test conditions lack consistency. In this way of proceeding, it is not possible to determine the cylinder axis independently of its power or to guarantee the independence of the two cylinder components that we are trying to determine. This is one of the accuracy limitations in "traditional" refraction when it comes to searching for the axis.

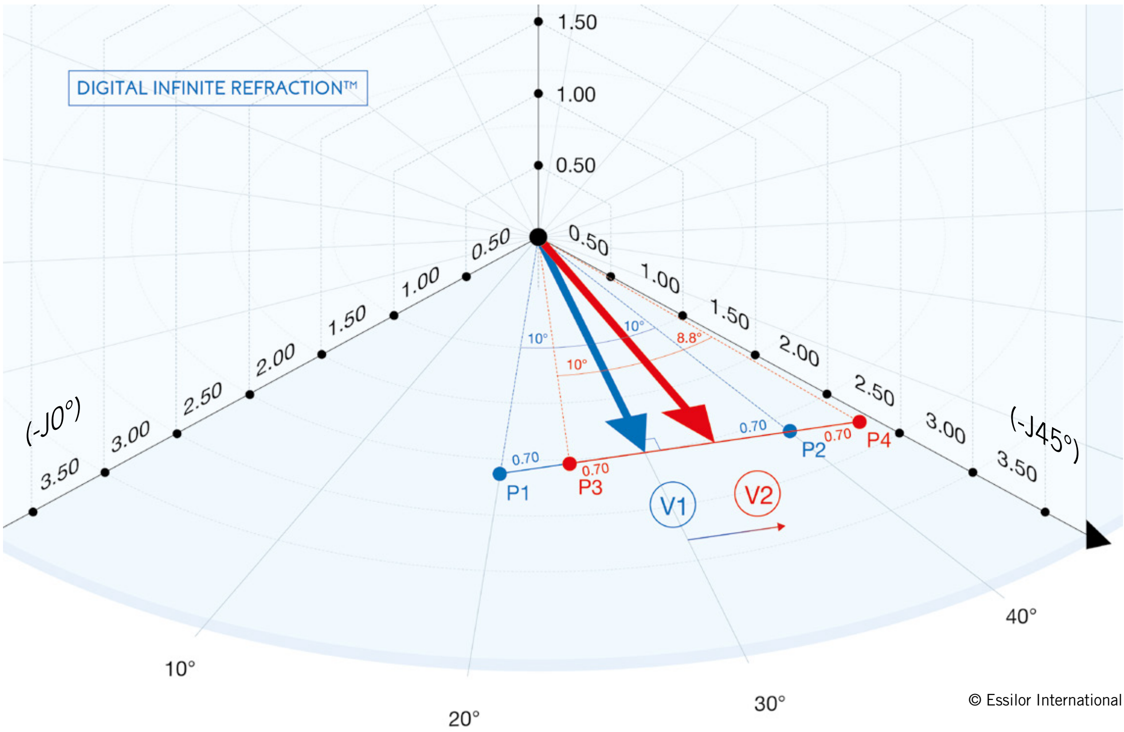


Figure 4: Cylinder axis test using the "Digital Infinite Refraction™" technique

3) <u>The dioptric increment used to determine the cylinder axis varies according to the cylinder power and is not consistent with the one used to determine the cylinder power.</u> The rotation effected between one axis test direction and another is left to the discretion of the practitioner. In practice, it is often constant, for example 5°, and not adjusted according to the cylinder power. Its dioptric effect, which is to say the translation in optical power of the axis rotation, is therefore variable and translates to the use of dioptric increments that vary according to cylinder power (see Table 3 on the dioptric effect of a cylinder axis rotation according to the cylinder power).
Moreover, these increments are not consistent with the dioptric increment used for changes in cylinder power, which is (-0.25) D. For the patient, this leads to a lack of uniformity in the effects of perception between the searches for the cylinder axis and cylinder power. As a result, the precision obtained in determining the axis is rarely equivalent to that obtained for the power and is often inferior to it. Once the cylinder power exceeds 1.25 D, a 5° rotation produces a dioptric effect superior to 0.25 D (see Table 4 presenting the cylinder axis rotation for creating a constant optical effect). This is the accuracy limitation found in the Jackson cross-cylinder method as implemented in the "traditional" refraction technique.
Ideally, to achieve full uniformity in patient perceptions, the axis rotation increment would need to be adjusted according to the cylinder power value so that it corresponds to constant dioptric effects (Table 4). Although experienced practitioners are skilled at rotating the axis according to the power, it is not possible to keep this dioptric increment rigorously constant. As we shall see, the vectorial technique for determining the cylinder, combined with the optical module with continuous power changes, makes it possible to keep this dioptric increment exactly constant and therefore to ensure complete consistency in patient perceptions.

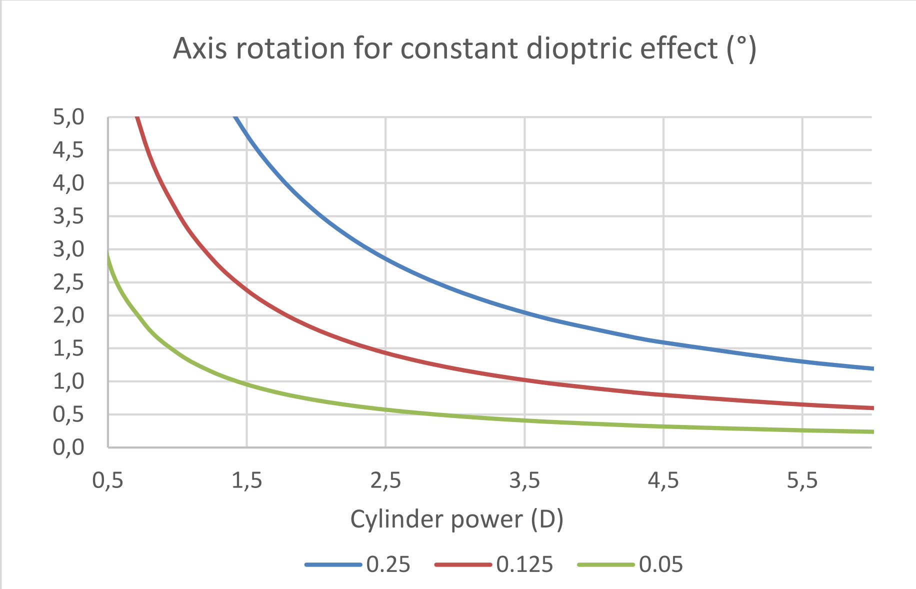
Table 3: Dioptric effect of a cylinder axis rotation. According to the cylinder power and for 3 axis rotations: 5°, 2° and 1°.

Tested Cylinder Power (Diopters)	Cylinder Axis Rotation (Degrees)		
	5°	2°	1°
0.50	0.09	0.03	0.02
1.00	0.17	0.07	0.03
1.50	0.26	0.10	0.05
2.00	0.35	0.14	0.07
2.50	0.44	0.17	0.09
3.00	0.52	0.21	0.10
3.50	0.61	0.24	0.12

For example, a 1° rotation in the axis of a 1.50 D cylinder has a dioptric effect of 0.05 D. If this same cylinder is turned 2°, the effect is 0.10 D and if it is turned 5°, the effect is 0.26 D.

– In the representation shown in Figure 3, a search for the axis using the "traditional" technique translates graphically to the fact that the dimension of Vector 2 (in red) is identical to that of Vector 1 (in blue) rather than being a projection of it, and that the test direction of Vector 2, made perpendicularly, is different from the direction tested for Vector 1. Thus, throughout the process of determining the cylinder axis in the traditional method, the cylinder power remains identical regardless of its orientation, the axis system of reference varies for each orientation of the cylinder and the direction changes made during the cylinder search do not maintain uniformity of perception for the patient. These permanent changes inevitably introduce a bias and are a source of inaccuracy, constituting an intrinsic limitation to precision in the "traditional" technique for determining the cylinder axis.

Table 4: Cylinder axis rotations producing a constant dioptric effect
Depending on the cylinder power and for 3 dioptric effect values (0.25 D, 0.125 D and 0.05 D).



A 5° cylinder axis rotation corresponds to a dioptric effect above 0.25 D as soon as the cylinder power exceeds 1.25 D. Starting with a cylinder power of 3.50 D, the axis rotation should be less than 2° to respect a 0.25 D increment (see the example in the figure): it should be 1° for a 0.125 D effect! The traditional refraction method, in which the axis rotation increment is most often constant in degrees, does not allow the practitioner to keep the dioptric change increment constant during the search for the cylinder axis. The vector method used in "Digital Infinite Refraction™" does make this possible.

With the "Digital Infinite Refraction™" technique

The digital cylinder axis search technique, which is made possible by phoropters with continuous power changes(*), uses a principle that is similar to the Jackson cross-cylinder method but with several fundamental differences:

1) No cross cylinders are physically present in the phoropter, but optical effects of virtual cross-cylinders are generated in the optical module, as previously explained.

2) The power of the cross cylinder used can be chosen, and therefore varied, and can be configured in the cylinder search algorithm. In the example presented, it is +/-0.35 D, and therefore has the formula +0.35 (-0.70).

3) Any dioptric effect induced by a modification with the corrective cylinder axis is automatically adjusted in the cylinder power and, as a result, compensated in the sphere power. This adjustment is made very precisely in 0.01 D resolution in such a way that the cylinder axis test direction and spherical equivalent power are kept fully constant throughout the entire test. This is possible due to the properties of the optical module of the phoropter with continuous power changes,(*) which allows to very precisely and simultaneously vary the sphere, cylinder and axis. Thus, the corrective cylinder axis test is performed using an axial component with a constant direction, perpendicular to the initial cylinder direction, and independently of other refraction components, very precisely respecting their values.

If we consider the earlier example of an initial correction of +1.00 (- 2.00) 30°, the axis test begins in the same way as in the "traditional" method. A cross-cylinder power is tested perpendicularly to the direction of the vector representing the initial correction (see Figure 4). As this cross-cylinder has a higher value, namely +/- 0.35 D, the tested axis variations are greater than in the "traditional" method, which more often uses a cross-cylinder of +/- 0.25 D. In the example chosen, the formulas tested are +1.06 (-2.12) 20.4° for Position 1 and + 1.06 (-2.12) 39.6° for Position 2. We can see that for this first test, the tested axis directions are symmetrical with regard to the initial direction tested: -/+ 9.6°. The patient perceives greater differences than in the "traditional" method and can more easily indicate which of the two positions he/she prefers. Let us suppose that the patient prefers the second position and therefore "request" an axis greater than 30°. Next, the algorithm will rotate the corresponding cylinder axis, in this direction and because it is chosen in this way, to a translation of half of the value of the 0.70 D cross-cylinder in the test direction: i.e. 0.35 D.

A fundamental difference compared to the "traditional" method can be observed at this point: management of the refraction via vectorial components results in the dioptric effect of the corrective cylinder axis variation being corrected on the value of the new cylinder and its consecutive effect on the spherical equivalent power is also compensated in such a way that keeps it constant. In other words, rather than keeping an identical cylinder value, it is adjusted to allow a search for the cylinder axis – or, more specifically, the cylinder's axial component projected perpendicularly to the direction of the initial axis – independently of its effects on the other refraction components and thus to conserve the same test conditions. In our example, the new formula to test becomes +1.015 (-2.03) 35°, where we can observe that the cylinder power has been adjusted by (-0.03) D and the sphere power has been compensated in consequence by +0.015 D.

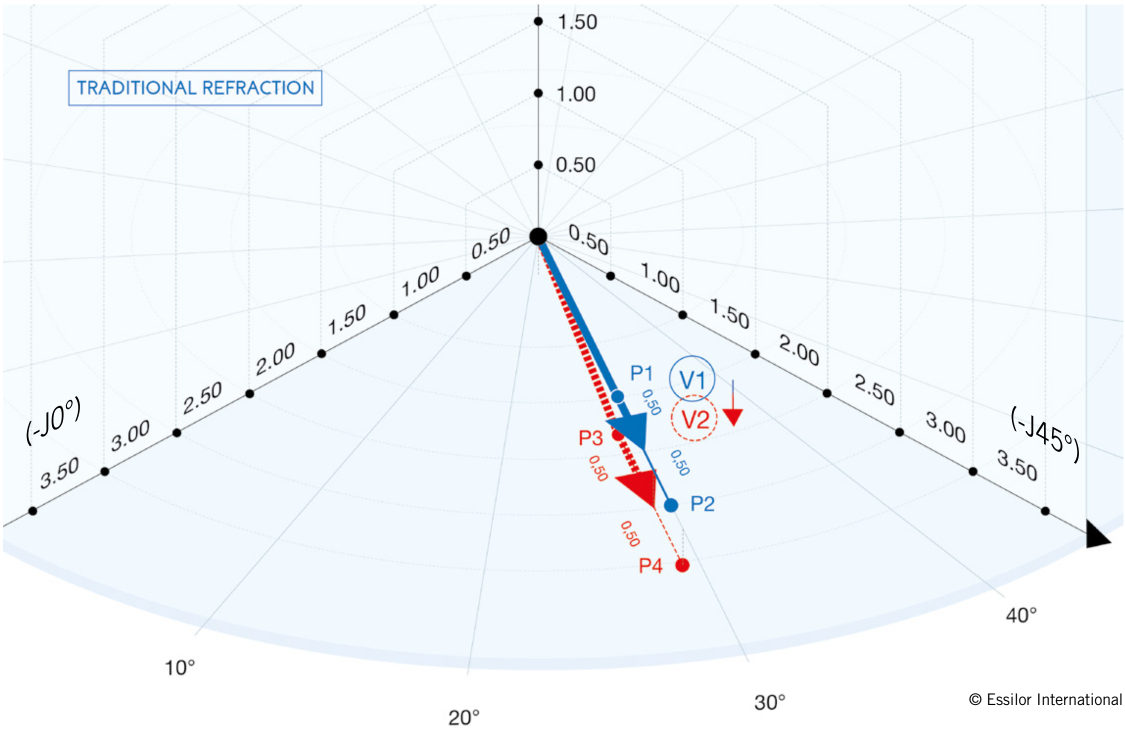


Figure 5: Cylinder power test using the "Traditional Refraction" technique

Graphically speaking, in Figure 4, searching for the cylinder axis using the digital method translates to the fact that the orthogonal projection of Vector 2 (in red) on the direction of the initial axis corresponds to Vector 1 (in blue) and the direction of the axis search on the J0° / J45° plane remains identical throughout the entire cylinder axis search process, which is to say perpendicular to the initial axis. Thus, the projection of the cylinder power determined according to the initial axis, which corresponds to the cylinder's second vector component, is respected and remains independent of the axial component. The cylinder axis search system of reference is thus kept constant. From a practical point of view, this is why, when the automated cylinder search algorithm is being used, the sphere, cylinder and axis all vary at the same time during any test of the cylinder's axial component.

As previously mentioned, another fundamental difference between the "traditional" and "digital" methods is that the axis modification increment can be chosen so that it is dioptrically identical to the one used to search for the cylinder power. More specifically, the dioptric effect of the axis rotation between two tested axis positions can be exactly the same as the one used during the changes made between two tested cylinder powers, as we will see later on. In our example, a choice has been made to use 0.35 D change increments, corresponding to half of the virtual cross cylinder power of +/- 0.35 D, at least at the beginning, both for the axis orientation changes and for the power changes. The dioptric effects produced during the axis and power searches are consistent and the patient's perceptions of them are uniform. This is an undeniable advantage of the "digital" technique, since it cannot be obtained in the "traditional" technique.

The following axis test is then performed in the same direction as the first test, with an identical cross-cylinder value (although it could be different), tested on either side of the direction of the new cylinder but, this time, with different angular values rather than equal ones as is done in the "traditional" method, in such a way that the dioptric increment is kept constant. In our example, again with a +/- 0.35 D cross-cylinder, the new formulas tested become +1.13 (-2.26) 25.0° for Position 3 and +1.02 (-2.04) 43.8° for Position 4. We can see that they are asymmetrical compared to the tested formula, both in axes and powers, rather than symmetrical as in the "traditional" method. This is what makes it possible to maintain the projection of the axial vector component in a constant direction. And this can be seen quite clearly when we compare Figures 3 and 4.

The search for the cylinder axis proceeds in this way until an inversion in the patient's answers is reached. In other words, the patient either asks for the axis value to be reduced after asking for it to be increased, or vice versa.

Later on we will take a closer look at the way the patient's answers are taken into consideration and the method of evaluating the final refraction value.

b) Cylinder power test:

- ***With "traditional refraction" technique:***

The most common traditional technique for verifying the power of a corrective cylinder involves using a Jackson cross-cylinder to determine whether the cylinder power should be increased or reduced. To do this, the practitioner orients the cross-cylinder in front of the correction in place by positioning its main meridians so that they are in correspondence with the corrective cylinder axis (in other words, by turning the cross-cylinder 45° compared to the orientation previously used to verify the corrective cylinder axis). They present the cross-cylinder in an initial position,

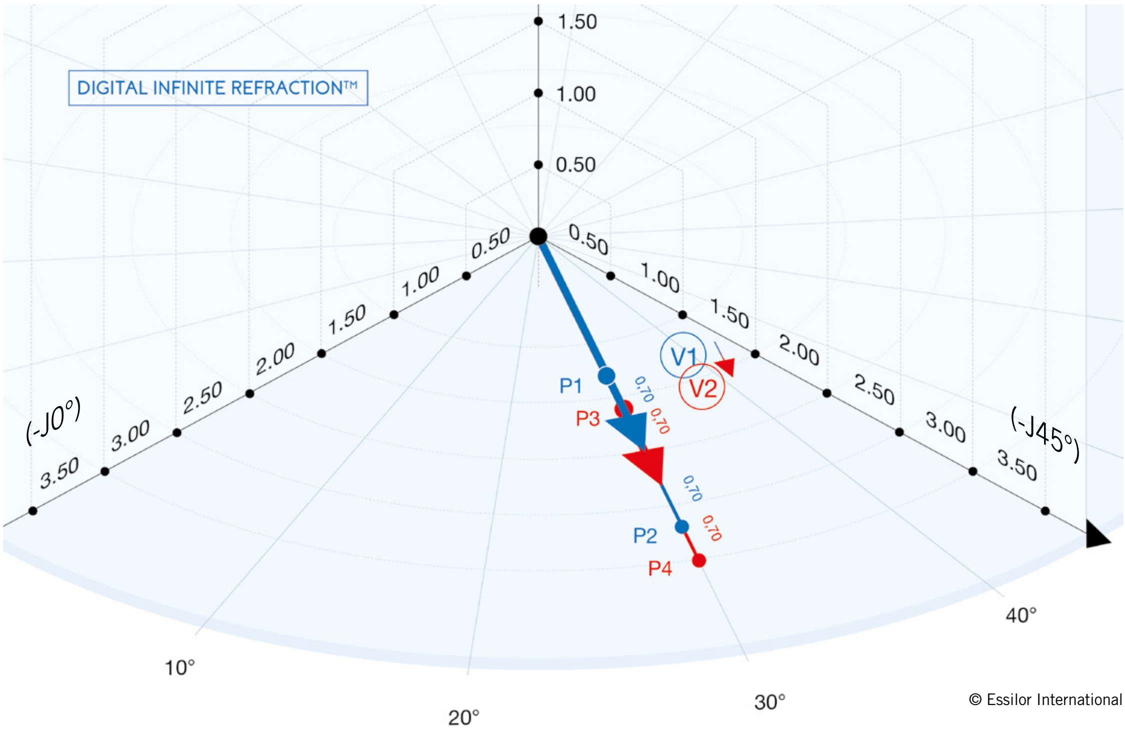


Figure 6: Cylinder power test using the "Digital Infinite Refraction™" technique

then rapidly turn it over and ask the patient to indicate in which position their vision is clearest (or, more specifically, the least blurry). When the lens is turned over, the + and - axes of the cross-cylinder are inversed and the corrective cylinder power is increased in one position and reduced in the other, without any effect on the mean sphere since the spherical equivalent power of the cross-cylinder is null. Let us suppose that they are using a +/- 0.25 cross-cylinder, and therefore a 0.50 D cylinder. The test described consists of increasing and reducing the cylinder by 0.50 D and asking the patient which he/she prefers.

Let us return to the earlier example of a prescription of +1.00 (-2.00) 30°, testing for cylinder power (Figure 5). The practitioner positions the cross-cylinder with its cylinder axis oriented according to the corrective axis of 30° and turns it over to test the following two positions: for example, Position 1, with the positive axis oriented at 30° and Position 2, with the negative axis oriented at 30°. In other words, the practitioner asks the patient if he/she wants the negative cylinder to be reduced in Position 1 or increased in Position 2. The power combination formulas – for the corrective cylinder and cross-cylinder – that are tested are as follows: +0.75 (-1.50) 30° in Position 1 and +1.25 (-2.50) 30° in Position 2. Let us imagine that the patient wants the cylinder power increased and therefore prefers Position 2. The practitioner then increases the cylinder power by (-0.25) D, according to the minimum increment available in traditional phoropters, which happens to correspond to half of the cross-cylinder power (without there necessarily being any relationship between the two). They then repeat the process. Next, they test the cylinder power of the new correction, with the formula +1.00 (-2.25) 30°, with two options: one reducing the cylinder power by 0.50 D and the other increasing it the same amount, with the formulas +0.75 (-1.75) 30° for Position 3 and +1.25 (-2.75) 30° for Position 4. They continue in this way until the patient no longer sees any difference between the two positions of the cross-cylinder or an inversion is reached in their answers. In other words, the patient asks either for the cylinder power to be reduced after asking for it to be increased or vice versa.

We can make an observation at this point: each time the cylinder power is modified, an undesirable effect is inevitably produced in the spherical equivalent power of the refractive formula, making a sphere adjustment necessary. In the example suggested, if the first correction tested, +1.00 (-2.00) 30°, has a plano spherical equivalent, the second correction tested, +1.00 (-2.25) 30°, will have a spherical equivalent of -0.12 D. A deviation of the spherical equivalent power is thus produced with each modification to the cylinder power. To be able to test the cylinder power independently of the other refraction components, one needs to be able to immediately compensate for the effect induced on the sphere. This is unfortunately impossible with traditional phoropters using lenses in 0.25 D increments. And it is generally only after a modification of (0.50) D to the cylinder power that the mean sphere power can be adjusted by an opposite half-value. Thus, most often, a +0.25 D sphere adjustment is made after each (-0.50) D cylinder is added to the corrective cylinder power. This happens automatically in motorised phoropters.

In the representation of the "dioptric space" (Figure 5), the traditional technique for testing cylinder power can be seen with a reduction (Position 1) or increase (Position 2) in the tested cylinder power. The increase in cylinder requested by the patient translates to an increase in the dimension of Vector V2 (in red) compared to that of Vector V1 (in blue) but, at the same time, by a change in the average power that makes Vector V2 no longer be located on the J0° / J45° plane but on a plane below it. Thus, the J0° / J45° plane for the cylinder search changes with each modification to the cylinder power rather than remaining constant. This is where we find another of the limitations of the "traditional" refraction technique, in which the effects of the cylinder on the sphere cannot be controlled with precision.

With "Digital Infinite Refraction™" technique

The "digital" cylinder power test technique is similar to the "traditional" technique using Jackson cross-cylinders but, as we have already seen, with the following three basic differences:

1) <u>The optical effects of the cross-cylinders are produced by calculation in the optical module</u>, in combination with the existing correction, as explained previously.

2) <u>The value of the cross cylinder used differs from that of the traditional +/- 0.25</u> and a greater power is used to facilitate the patient's answers. In the example at hand, the value of the cross-cylinder used is +/- 0.35 D, or a formula of +0.35 (-0.70).

3) <u>Any modification to the cylinder power is simultaneously accompanied by an adjustment to the sphere power to keep the spherical equivalent power constant</u>, with a resolution of 0.01 D. Thus, for any modification of (-0.02) D to the cylinder power, an increase of +0.01 D is automatically made to the sphere power.

Let us again consider our example of a +1.00 (- 2.00) 30° correction in which, this time, we want to verify the cylinder power (Figure 6). Again, using the cross-cylinder technique, the idea is to see whether it should be increased or reduced. Since the virtual cross-cylinder power is +/- 0.35 D, the algorithm introduces a 0.70 D cylinder variation, reducing (Position 1) or increasing (Position 2) the existing corrective cylinder. The following refractive formulas are tested: +0.65 (-1.30) 30° in Position 1 and +1.35 (-2.70) 30° in Position 2. We see that the sphere power is automatically adjusted by the opposite half of the cylinder variation introduced, which is also the case in the "traditional" technique. Let us suppose that the patient wants the cylinder increased and therefore prefers Position 2. The algorithm would then modify the value of the corrective cylinder by half of the variation of 0.70 D tested, or 0.35 D, for example, because the value of the increment could be chosen differently. But at this point, at the same time as the corrective cylinder is modified, the power of the sphere is also compensated in order to keep the spherical equivalent power constant. The new correction tested would thus become +1.17 (-2.35) 30°. Note the +0.17 D sphere adjustment, which could not be done in the traditional technique. It would then look for the cylinder power, testing two new powers whose formulas are +0.82 (-1.65) 30° for Position 3 and +1.52 (-3.05) 30° for Position 4. It would then continue in this way until an inversion in the patient's answers is reached, adjusting the sphere power for each modification to the cylinder power.

Graphically speaking, in the representation of the dioptric space (Figure 6), the "digital" cylinder power test technique can be seen, as for the "traditional" technique by a reduction (Position 1) or an increase (Position 2) in the cylinder power suggested, and therefore a proposal to shorten or lengthen the length of Vector V1 (in blue). The increase in cylinder power requested by the patient translates to a lengthening of the dimension of the vector from V1 (in blue) to V2 (in red) but, this time, with a fundamental difference: the mean sphere power is kept constant via its simultaneous adjustment when the cylinder power is increased. In practice, Vector V2 remains on the same plane and the search for the cylinder continues on a single J0° / J45° plane, keeping all other characteristics constant. This is a major difference and a clear advantage with the "Digital Infinite Refraction™" compared to "Traditional Refraction" when it comes to determining a patient's corrective cylinder.

We will continue the presentation and discussion of this topic in a third and last article that will follow.

KEY INFORMATION:

- In the "traditional" refraction method:
  - the cylinder axis is determined on the basis of the current direction of the axis, with dioptrically variable increments in axis rotation,
  - the cylinder power is determined while mean sphere power is varying,

  in other words, under conditions that change throughout the process of determining the cylinder.

- In the new "digital" refraction method:
  - the cylinder axis is determined on the basis of a fixed direction, with dioptrically constant increments of axis rotation,
  - the cylinder power is determined while maintaining the mean sphere power constant,

  in other words, under fixed, consistent conditions throughout the process of determining the cylinder.

- Thus, the testing technique used in new "Digital Infinite Refraction™" allows for a more precise determination of the corrective cylinder.

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SUBJECTIVE REFRACTION: A NEW VECTORIAL METHOD FOR DETERMINING THE CYLINDER (3/3)

The refraction technique traditionally used to determine the corrective cylinder for a prescription has changed very little over the years, mainly due to the limitations imposed by subjective phoropters, which present lenses in increments usually no smaller than 0.25 D.
Today, thanks to phoropters with continuous power changes(*) that allow to simultaneously and accurately act on sphere, cylinder and axis, it is now possible to develop new refraction techniques.
This series of three articles describes the principles of a new vectorial method for determining the corrective cylinder and presents the rationale for an associated automated cylinder search algorithm.

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Trained in France as an optician and optometrist, Dominique Meslin has spent almost all his professional career with Essilor. He started in Research and Development department working on physiological optics studies and then moved to several Technical Marketing and Communication positions for Essilor International, in France and also in the USA. For more than 10 years, Dominique was the Director of Essilor Academy Europe. He then focused on Professional Affairs activities for Essilor Europe. He is now in charge of the new Refraction Solutions for the Instruments Division of Essilor International. All over his career Dominique has conducted many training seminars for the Eye Care Practitioners. He is the author of several scientific papers and many Essilor technical publications, including the "Ophthalmic Optics Files" series.

KEYWORDS
Subjective refraction, vectorial refraction, dioptric space, cylinder search, cross cylinders, phoropter, refraction algorithm, Vision-R™ 800.

Following the last two articles (published by Points de Vue in November and December 2020), we now continue our overview and discussion of our new vectorial-based cylinder-determination method. In this third and final article, we compare the techniques used in "traditional" and "digital" refraction to find cylinder values and look at how each one is charted in the dioptric space. We then discuss the advantages and outlook offered by the new "Digital Infinite Refraction™" method.

4) The traditional refraction method for determining the cylinder: an indirect process with permanent system of reference changes that limit accuracy.

As previously explained, the traditional method for determining the cylinder consists of searching first for the cylinder axis, then the cylinder power and finally adjusting the sphere power. Each of these components must be tested a number of times.

To determine the cylinder axis, the practitioner searches for its direction in increments (for example 5°) between each axis direction tested until the final axis orientation is found, in the event of an equal blur perception in the two cross cylinder positions, or until the axis orientation in a 5° angle is established, in the event of an inversion in the patient's answers.

And to determine the cylinder power, practitioners search for its value by increasing (or reducing) it in increments of (-0.25) D until they find the exact value if there is an equal blur for both positions of the cross cylinder or to establish its value between two increments of (-0.25) D if there is an inversion in the patient's answers.

Let us again consider our example of a starting refraction formula of +1.00 (-2.00) 30°, represented by an initial vector located on the J0° / J45° plane (see Figure 7), and see how the refraction is found. The first steps

(*) Vision–R™ 800 phoropter with smooth power changes by Essilor Instruments involve looking for the cylinder axis, carrying out successive axis tests (turning over a cross cylinder whose handle is oriented according to the direction of the axis to be tested) and taking into account the patient's answers. For example, the practitioner tests the 30° direction (1), which the patient asks to have increased, then the 35° direction (2), which they also ask to have increased and, finally, the 40° direction (3), which they ask to have reduced. The practitioner then chooses an orientation between the last two directions tested, for example 38° (4), for which it proves, after another testing, that the patient no longer perceives any difference between the two positions of the cross cylinder. The axis found is therefore 38°.

Next, the practitioner searches for the cylinder power, carrying out several successive tests in which cylinder powers are increased (or decreased) in increments of (-0.25) D (turning over a cross cylinder whose main meridians correspond to the cylinder axis to be tested) and taking into account the patient's answers. For example, the practitioner thus tests the powers that the patient successively asks to have increased – (-200) D (4), then (-2.25) D (5) and finally (-2,50) D (6) and then the power (-2.75) D (7) that he asks to have reduced. The practitioner reduces the cylinder power (-0.25) D and since (-0.50) D has been added to the initial corrective cylinder, they adjust the sphere +0.25 D, reaching the final formula of + 1.25 (-2.50) 38°.

Graphically speaking, this cylinder search translates in the dioptric space to the fact that:

- The first steps – (1), (2), (3) and (4) – in the cylinder axis search take place in the J0° / J45° plane along a "circular" line of constant cylinder power of (-2.00) D, leading to an axis location between 35° and 40° and found, in this example, at 38°.
- The following steps – (4), (5), (6) and (7) – in the cylinder power search take place along a constant axis direction (38°) as the power is increased, which is to say radially moving away from the system of reference's origin. With this increase in cylinder power, the spherical equivalent power (or average power) decreases, which translates graphically to the fact that points (5), (6) and (7) progressively "sink" below the J0° / J45° plane as the cylinder power increases.
- The last step (8) in the cylinder reduction and final sphere adjustment involves a radial reduction in cylinder power and an adjustment toward the convex of the sphere power and therefore by a raising of point (8) on the J0° / J45° plane (a +0.25 D compensation of the sphere after an increase of (-0.50) in the cylinder).

At this point, we can make the following observations:

- The process used in the traditional technique for cylinder search appears to be quite indirect. This can be clearly seen, in Figure 7, by the way in which the dioptric space is explored: first in a "circular" way for the axis search, and then "radially" for the power search, with an effect on the "altitude" which is then compensated for. Remember that this exploration method is directly linked to the limitations imposed by traditional phoropters and, more particularly, the fact that action on the sphere, cylinder axis and cylinder power can only ever be performed separately and in increments of 0.25 D.
- The cylinder search using the traditional technique takes place in a system of reference that is modified with each of the patient's answers: during a cylinder axis change because the cylinder power has not been adjusted after a modification to the axis and also during

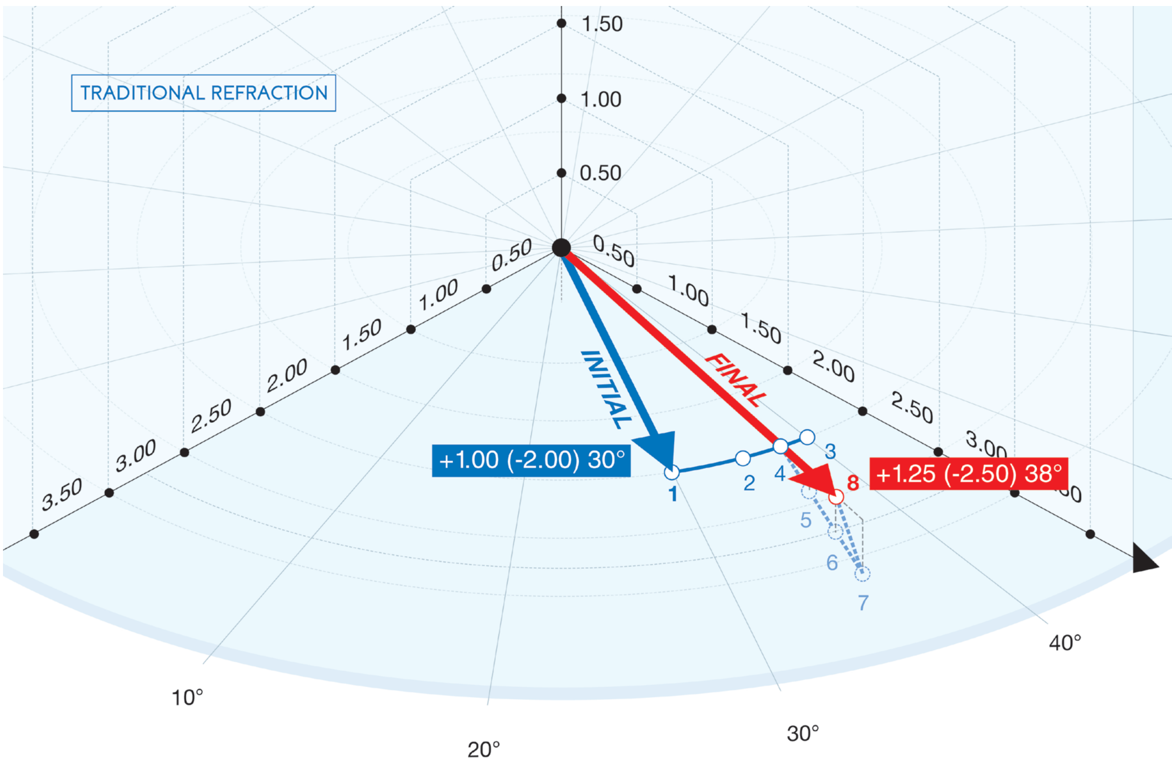

Figure 7: Determining the cylinder using the "Traditional Refraction" method:
Cylinder axis then cylinder power, followed by a sphere adjustment.

a cylinder power change because the sphere power has not been adjusted to keep the spherical equivalent power constant. The practitioner must determine the cylinder axis and power separately, although they are intimately linked in their polar expression and any modification to one of them inevitably has an influence on the patient's perception of the other. Thus, during the cylinder axis search, each axis test performed – as we have seen in Part 3a of this publication – perpendicularly to its direction is made in an orientation that changes with each modification to the cylinder axis. During the power search, each test is carried out with a variable spherical equivalent power. As a result, the cylinder search system of reference "moves" over the course of the cylinder search and makes its determination less accurate. It should also be noted that there is variability and potential inconsistency in the patient's answers.

- During the cylinder search, the power and axis variation increments most often remain constant: a 0.25 D increment for the power, as imposed by the lenses available in traditional phoropters, and a 5° increment for the axis, as is offered by default by the phoropter or chosen by the practitioner even when other options are possible. It should be noted that these increments are generally the same regardless of the refraction value, whether it is high or low, and regardless of the patient's sensitivity to dioptric changes. Let us also note that these increments are often greater than the patients' dioptric sensitivity. In one study, it was found to be under 0.25 D for 95% of the patients and under 0.125 D for 44% of them, or nearly one of every two patients.[7]

- In the Jackson cross-cylinder method, we compare the blurred and variously distorted vision of points (or optotypes) that are not always easy for the patient to evaluate and may consequently require repeated tests. The cross-cylinder value of +/- 0.25 D generally used can prove insufficient in generating differences that are significant enough to be perceived by the patient. The search for the cylinder axis, like that for the cylinder power, ends when an equally blurry vision is found for both positions of the cross cylinder, which can be difficult for the patient to evaluate and can prove troubling to them. It may indeed seem strange to them that the cylinder search stops when their vision is still blurry in all directions.

- In the traditional cylinder search process, the practitioner's experience is essential. This is because mastery of the Jackson cross-cylinder technique requires a lot of practice. The practitioner must evaluate and interpret each of the patient's answers to perform the cylinder search and make decisions, for example to modify the cylinder axis direction in the direction requested by the patient, and to stop the cylinder axis search to begin the power search, or to consider that the cylinder search is finished. Furthermore, the practitioner has to simultaneously perform a refraction and draw up a prescription, which is to say interpreting the results to decide upon the prescription: for example, under-correction of the cylinder power or sphere, or a moderation in the change of axis or the decision to stop searching for the cylinder. For these reasons, a refraction result can depend on the practitioner who performs it, which means there is inevitably a certain amount of variability. Traditional subjective refraction can therefore be viewed as doubly subjective, since it depends on both the patient's evaluation and the practitioner's interpretation!

It is clear that the traditional refraction method for determining the cylinder has intrinsic limitations, both in terms of consistency in the patient's answers and accuracy of the dioptric increments used. It cannot be used to determine the refraction with enough accuracy to match patients' true dioptric sensitivity.

2) Determining the cylinder with Digital Infinite Refraction™: a direct process with a consistent system of reference and two iterations to ensure great precision.

The digital refraction method for finding the cylinder involves – according to the choice that has been made – searching first for the cylinder power component according to the initial refraction axis direction and then the cylinder axis component along a direction perpendicular to this initial axis. This is made possible by vector management of the cylinder components, which induces an adjustment to the cylinder power with each modification to its axis. The cylinder search is also performed with a constant spherical equivalent power throughout the entire process. This search is based on two new principles:

- First, the cylinder power component is always tested in the direction of the axis of the initial refraction – or in a parallel direction in the dioptric space – and the cylinder axis component is always tested in a direction perpendicular to the direction of the initial axis. The search for the power and axis components is performed in two fixed directions and independently of each other.
- Second, an inversion in the patient's answers is always be sought – rather than equal answers as in the traditional method – and a statistical estimate of the most probable value of each power and axis component is made for all of the answers given by the patient, rather than (in the traditional method) the practitioner making a decision, for axis and then power, according to their evaluation of the patient's last answer.

The search for the power component begins according to the initial axis with an increase – or reduction – in power by one increment, chosen but configurable, of (-0.35) D until an initial inversion in the patient's answers is obtained. During this phase, the axis remains fixed and only the cylinder power varies with a corresponding adjustment to the sphere value. An initial power value is obtained in this way, midway between the last two cylinder powers tested.

The search for the axis component then continues in a direction perpendicular to the initial axis direction, testing the axis variation effects induced by a 0.70 D variation on either side (using a virtual cross cylinder of +/- 0.35 D whose value was chosen but can be modified in the algorithm). With each of the patient's answers, the axis is modified as requested and in a direction that remains constant in the dioptric space with an adjustment to the cylinder power and the corresponding compensation to the sphere power. The second inversion in the patient's answers is then sought, relative to the axis direction this time. In this way, an initial axis value is determined midway between the last two directions tested – the ones giving rise to contradictory answers from the patient.

Let us take another look at our example of an in initial correction of +1.00 (-2.00) 30°, represented by an initial vector located on the J0° / J45° plane (Figure 8). The first steps involve carrying out several power tests (using the digital refraction technique described previously) in the 30° direction and according to the patient's answers. For example, the practitioner tests the powers that the patient asks to have increased: (-2.00) D (1) and then (-2.35) D (2), and finally the power he asks to have reduced (-2.70) D and thereby obtain an initial cylinder power estimate of (-2.52) D according to the 30° axis, which is the value midway between the last two powers tested. It should be noted that each power modification is always accompanied by a compensation in the sphere, of an opposite half-value, to keep the spherical equivalent power constant.

Next, the cylinder axis component search is performed in the direction perpendicular to the initial axis (using the digital refraction technique) and according to the patient's answers. The practitioner first tests the 30° axis (4), which the patient asks to have increased, then the 34° (5) axis, which he also asks to have increased, and finally the 38° axis (6), which he asks to have reduced. The inversion in answers sought is thus obtained and the final angle value is the one between the last two directions tested, or 36°. Note that for each axis variation, the cylinder power is adjusted and the sphere power is compensated as a result.

In this way, we reach the formula +1.24 (-2.49) 36°, after an inversion in answers according to the initial axis correction and an inversion in answers according to the perpendicular direction.

Graphically speaking, this translates in the dioptric space to the fact that:

- The first steps – (1), (2), (3) and (4) – in the cylinder power component search are made in the 30° direction and remain located on the J0° / J45° plane, unlike what happens in traditional refraction, in which they progressively shift along the J0° / J45° plane.
- The following steps – (4), (5), (6) and (7) – in the cylinder axis component search take place in a direction perpendicular to the direction of the initial axis, starting from point (4). These points are aligned according to the same straight line, which mediates the segment linking points (2) and (3) – which is to say the perpendicular line in its middle – rather than along a circle centered on the origin of the system of reference corresponding to the initial cylinder power as in traditional refraction. These points are all rigorously located on the J0° / J45° plane, in which the entire cylinder search is performed.

Several observations can be made here:

- <u>The digital method for determining the cylinder is much more direct than the traditional one.</u> Its representation in the dioptric space (Figure 8) shows that the search begins radially, according to the initial cylinder axis, to find the cylinder power component, and then along a line perpendicular to the direction of the initial axis to look for the cylinder axis component, always remaining on the J0° / J45° plane. This is due to the fact that, as explained previously, any variation in the cylinder power is automatically compensated for in the sphere power and, thanks to the management of vectorial components, any modification to the axis brings about an adjustment in the cylinder power and therefore a compensation in sphere power.

- <u>The system of reference in which the cylinder search is performed remains constant throughout the search</u>: the power and axis tests are always done in the same respective perpendicular directions: the initial cylinder axis direction for the power search and its perpendicular direction for the axis search. In other words, the cylinder search is performed according to its vectorial components in constant directions. This vectorial technique allows the practitioner to maintain greater consistency in the patient's answers and to increase precision in determining the cylinder. It offers the possibility of cumulating the patient's answers according to two constant directions and estimating the cylinder power and axis statistically, rather than basing them on the patient's final single answer as is the case in traditional refraction. We will look at this again later.

- <u>The cylinder search becomes more accurate during the refraction process</u>:
  - The cylinder power modification increment is initially higher in the digital method than in the traditional one – (0.35) D rather than (0.25) D – and this allows for a more rapid power search and facilitates the patient's answers. This increment is then progressively reduced. It is halved after the first inversion in the patient's answers and then fine-tuned further. It may also be increased again if the patient's answers are inconsistent. Note that in traditional refraction, this increment would remain constant at 0.25 D throughout the entire process.
  - The cylinder axis modification increment is dioptrically constant and identical to the one used to search for the cylinder power: 0.35 D to start with. The advantage of this is that it creates axis changes which, translated to diopters, generate variations in uniform perceptions for the patient compared to those perceived in the power search. This increment is then reduced by half upon the first inversion in the patient's answers – relative to the axis test – and will then be fine-tuned further. It may also be increased again if the patient's answers are inconsistent.

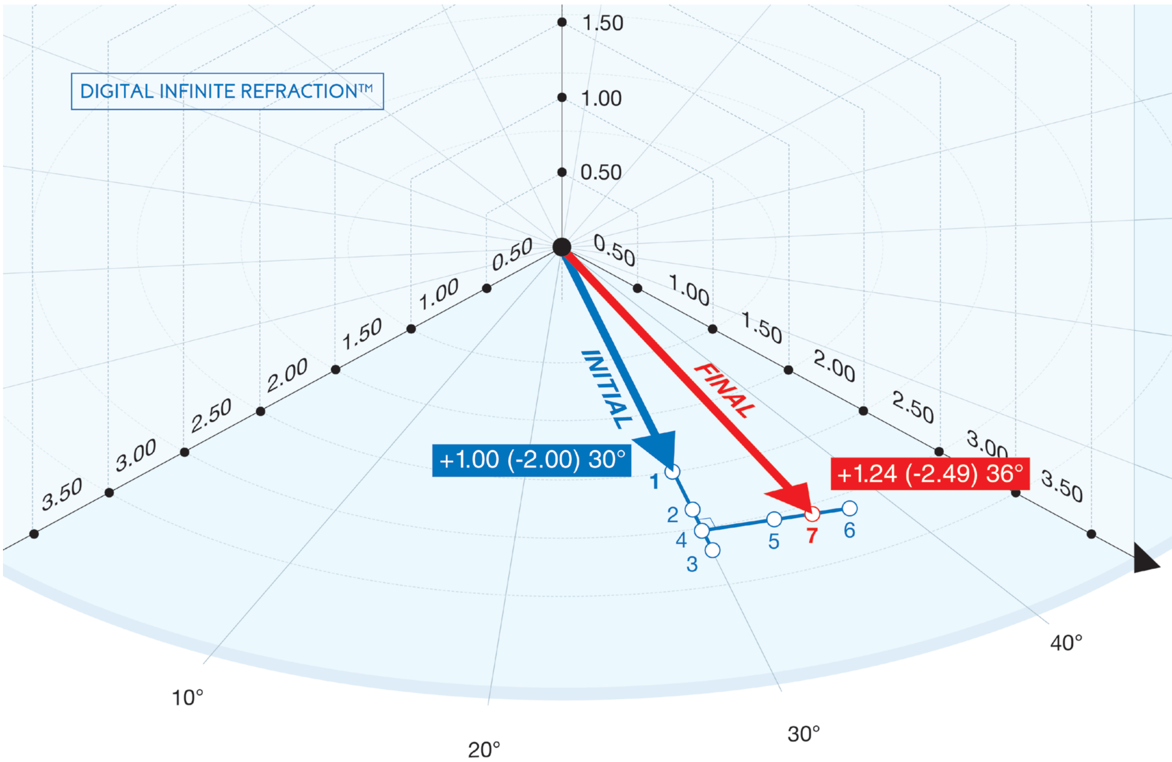

Figure 8: Determining the cylinder using the "Digital Infinite Refraction™" method:
cylinder power then cylinder axis, with simultaneous compensation of cylinder power
and a constant spherical equivalent power

Inversely, in traditional refraction, the axis variation increment is chosen by the practitioner and is often 5° regardless of the cylinder value. This has two noteworthy consequences: first, the dioptric effect produced by the axis cylinder change can vary from one patient to another since it depends on the cylinder power and second, this effect is not uniform with the one used to search for the power. This applies to all patients. In other words, the fact that the axis modification increment is chosen by the practitioner and can be constant angularly makes it very dioptrically variable!

- In digital refraction, the cylinder power and axis components are evaluated by establishing the value sought and on the basis of inversions in the patient's answers. They are estimated independently of each other and in a statistical way over all of the patient's answers. We will look at this again later.

  In traditional refraction, the cylinder power and axis values are, to the contrary, established according to the patient's final answer – once they indicate that two positions have the same blurriness – and a decision made by the practitioner. It is therefore inevitable that the process involves a certain amount of subjectivity, which means variability.

With the Digital Infinite Refraction™ method, which uses an automated algorithm, the cylinder search is conducted independently of the practitioner's techniques and decisions. It involves determining the patient's refraction value, which the practitioner will then interpret and modify to draw up the prescription.

A second cylinder search iteration for greater precision

In the Digital Infinite Refraction™ approach, the cylinder search does not consist of a single determination of the cylinder as it does in the traditional technique. Rather, the search algorithm* offers a second verification of the cylinder power and axis after the first search. The idea is to fine-tune the refraction found in the first iteration by searching (in the same directions in the dioptric space) for two new inversions in the patient's answers. When this is done, all the answers are accumulated according to the two fixed directions – the cylinder power and axis components – to carry out a statistical analysis to evaluate the cylinder power and axis thresholds while simultaneously verifying the consistency of the patient's answers. Let us take a closer look at this second iteration.

After the first cylinder search, the following takes place:

- First, a second verification of the cylinder power component is performed, again in the direction of the initial 30° axis from our example, until a new inversion is observed in the patient's answers. This second evaluation complements, fine-tunes and confirms the first cylinder power evaluation already performed in this direction during the first iteration.

- In the same way, a second verification of the cylinder axis component is carried out, again perpendicularly to the initial axis direction, until there is a new inversion in the patient's answers. This is how a more precise evaluation of the axis can be performed.

- During each of these answer inversions, the dioptric increment of variation in power or axis is again reduced in order to fine-tune the search. Note that this increment is nevertheless kept at a level sufficient to be perceived by the patient but that it can be increased again if inconsistencies are observed in the patient's answers.

- For both cylinder components – power and axis – all of the patient's answers are taken into consideration and analysed statistically. More specifically, the answers are accumulated in the initial cylinder axis direction for the cylinder power component and in the direction that is perpendicular to it for the cylinder axis component. According to each of the directions that are determined, for all the answers the patient gives during the first and second iterations, the most probable value is estimated for the cylinder power and axis components. This is how the algorithm determines the thresholds for the cylinder power and axis, which represent an accurate refraction result. They are calculated using Cartesian coordinates and plotted with polar coordinates.

- The consistency of the patient's answers is verified throughout the search process, and this is what determines at which point the process should end. As a result, the more consistent the patient's answers are, the less time the cylinder search process takes. Inversely, if the patient's answers are inconsistent, it can take a while to achieve the reliability needed.

In this way, the practitioner reaches a final refraction of +1.21 (- 2.42) 35°.

Graphically speaking, this second iteration of the cylinder search translates in the dioptric space (see Figure 9) to the fact that:

- After steps (1) through (6) of the first iteration, which lead to the first refraction estimate, the examination continues with steps (7) through (12), in which the refraction is fine-tuned.

- The second verification of the cylinder power component – steps (7), (8) and (9) – takes place along a line parallel to the initial cylinder axis direction in the dioptric space: in Figure 9 we can see that the straight line joining points (7), (8) and (9) is parallel to the one joining points (1), (2) and (3). The initial direction used to search for the cylinder power component is kept constant while the axis component is adjusted to its most probable value after the first iteration.

- The second verification of the cylinder axis component – steps (10), (11) and (12) – is performed along a direction perpendicular to the cylinder's initial axis. In Figure 9 we can see that it is parallel to the right line joining points (4), (5) and (6). The initial direction used to search for the cylinder axis component is kept constant while the power component is adjusted to its most probable value after the second iteration.

- Another noteworthy detail is that unlike what happened during the first cylinder search iteration, point (7) is not found exactly midway between points (5) and (6), the points for which the inversion of the patient's answers arose, but is slightly offset. Similarly, we can see that point (10) is not midway between (8) and (9) but also a bit offset with respect to them. This comes from the fact that starting from the second cylinder search iteration, these points result from an estimate of the new "point" to be tested based on the patient's answers for the power and axis components, respectively. In other words, the answers already given by the patient during the first iteration are taken into consideration in the second iteration. The main advantage of the vectorial refraction technique is that one can examine the power and axis components independently of each other and accumulate answers according to these two directions to analyse them statistically and separately evaluate the most probable cylinder power and axis values.

Discussion:

Let us now discuss the advantages that Digital Infinite Refraction™ offers and the outlook for future developments that it makes possible:

- Consistency and accuracy: the vectorial approach used to search for the cylinder allows practitioners to carry out refraction with a consistent system of reference: on the one hand, the same dioptric system of reference is used throughout the search process and on the other, the dioptric effects produced during the searches for power and axis remain consistent. This technique allows to very accurately determine the cylinder, in 0.01 D and 1° increments, which was never possible until now.

- Psychometric methods: the new method allows us to implement threshold search techniques like the ones traditionally used in psychophysics. We are thus no longer limited to evaluating sphere, cylinder power and cylinder axis values as we are in traditional refraction, but rather look for the most probable threshold values for the three Cartesian components of refraction: spherical equivalent, the J0° horizontal cylinder component and the J45° oblique cylinder component, determining them statistically. The refraction thus becomes a truly physiological measurement!

- Precision in line with patients' true dioptric sensitivity: while traditional refraction using lenses in 0.25 D increments is not precise enough to match patients' real dioptric sensitivity (often less than 0.10 D), the new technique using 0.01 D optical increments makes it possible to determine refraction with a high degree of accuracy, the only limitation being the patients' dioptric sensitivity. It is no longer the phoropter that limits accuracy in refraction but the patient's sensitivity. Even better, the new technique also allows to evaluate the dioptric sensitivity of each patient during the refraction examination itself, thus offering a new complementary parameter to accompany the refraction result and help interpret it.

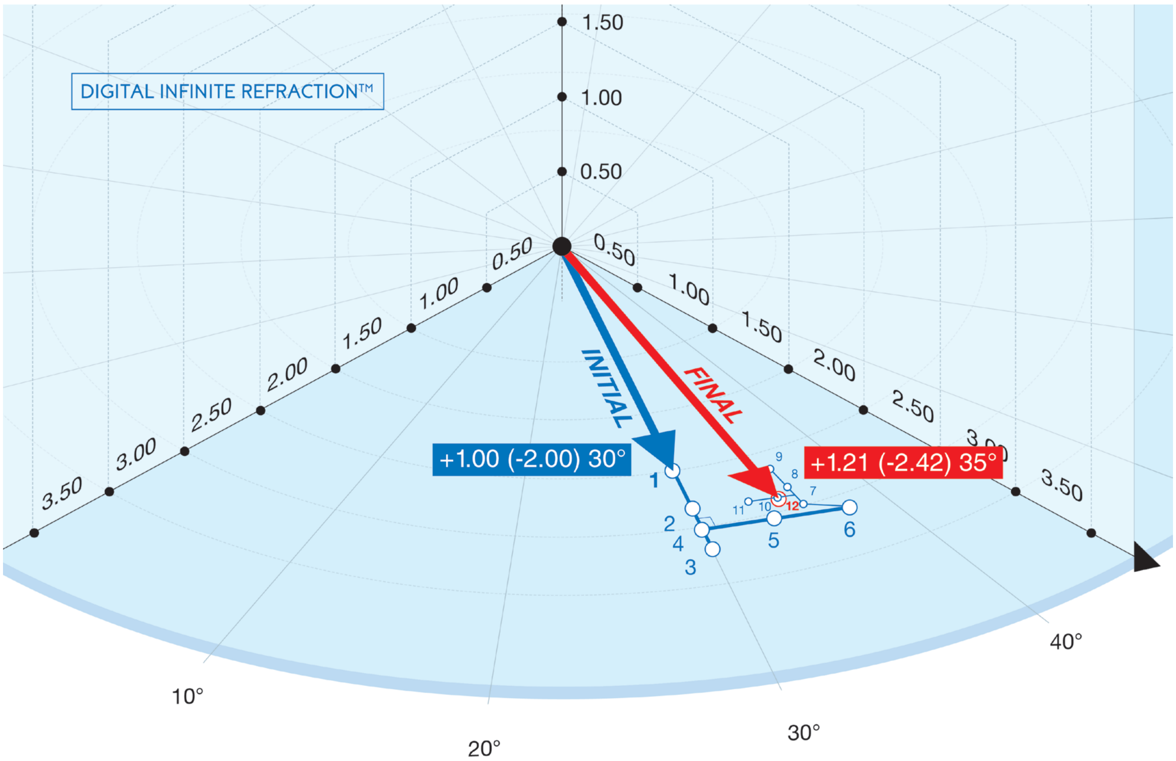

Figure 9: Determining the cylinder using the "Digital Infinite Refraction™" method

A double iteration of cylinder power and axis search in a single sequence

– Refraction-assistance algorithms: the formalisation of refraction techniques, in particular the one for the cylinder search, in automated refraction tests and programs allows practitioners to offer a certain standardisation in refraction examination methods. It should help eliminate the inevitable variability in practices and increase the reproducibility of refraction results from one practitioner to another, thereby making subjective refraction more objective!

– A new "refraction then prescription" sequence: the new approach also allows for a refraction sequence that differs from the traditional approach. Initially, this involves determining a subjective refraction value using the phoropter's algorithms and then in a second step, interpreting this result to draw up a prescription. Thus, unlike in the traditional simultaneous refraction-prescription process, which is to say interpreting the results during the refraction examination itself, with the subjectivity inherent in it, a new "refraction then prescription" sequence becomes possible. This new approach can help dissociate "refraction" from "prescription" and thus transform the way is refraction is viewed and performed.

– Corrective lenses in 0.01 D increments: although refraction can now be determined in 0.01 D increments, this accuracy clearly has no advantage unless corresponding optical corrections can also be offered. Happily, lenses are now available in 0.01 D increments thanks to the digital surfacing technology developed more than 10 years ago. Today, the greater accuracy now possible with the new phoropters allows us to offer patients high-precision corrections.

Conclusion:

Although subjective refraction methods have remained virtually unchanged for over a century, they are now undergoing a major change. The advent of phoropters offering continuous power changes has made new subjective refraction techniques possible and allowed us to rethink our approach to subjective refraction. As we have described in this series of three articles, the corrective cylinder search can be performed using a new vectorial technique that is both more consistent and more accurate. Similar approaches have also been developed for other refraction tests: a new technique for fogging and unfogging, an automated sphere determination, an exact binocular balance determination, an approach to and automated measurement of near-vision addition, etc. The rationale developed for each of these tests translates to algorithms implemented in the automated refraction tests. The succession of these various tests makes it possible to create refraction programs that the practitioner can use as is or personalise according to need.

Refraction tests and programs using the next generation of phoropters(*) are now available. With the total flexibility of the optical module and its controls, great scope for new refraction-related research and innovation is now opening up. In the future, even more innovative new tests, algorithms, programs, protocols and methods will emerge. And since these next-generation phoropters are connected tools, it will be possible to remotely update their software to integrate the latest advancements on a regular basis. Since the programs are also so easy to use and the phoropters are connected, the techniques can also transform the very way refraction is performed: delegated refraction, remote refraction and even self-refraction may become possible. We are clearly at the beginning of a revolution in refraction possibilities!

The technology of the new phoropters will make it possible to automate and standardise refraction techniques and determine patients' correction with greater accuracy. It will also make practitioners' day-to-day work easier, since they need simply monitor the automated processes, intervening as necessary. They must therefore thoroughly understand the workings of the algorithms involved in the refraction process. It is our hope that this publication has increased their understanding of the admittedly rather complex algorithm for vectorial-based corrective cylinder searches. But above all, we hope that it will promote the adoption and usage of automated cylinder determination processes by eye-care-professionals so that they can offer patients (even) more accurate optical correction of their astigmatism.

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KEY INFORMATION:

- In the traditional refraction method, the cylinder is determined in an indirect process with continually changing system of reference, which limits precision.
- In the digital refraction method, the cylinder is determined using a direct process with a consistent system of reference and double iteration to ensure a high level of precision.
- The new Digital Infinite Refraction™ method offers greater refraction accuracy thanks to its use of evolving automated algorithms that open up new possibilities in terms of making refraction exams easier.